ZIBELINE INTERNATIONAL™ PUBLISHING

ISSN: 2773-6202 (Online)
CODEN: JTIOCC

Journal of Technology & Innovation (JTIN)

DOI: http://doi.org/10.26480/jtin.02.2025.28.36

CrossMark

REVIEW ARTICLE

EVALUATING THE EFFICIENCY OF BOOTSTRAP-ENHANCED FGLS FOR ROBUST ECONOMETRIC INFERENCE UNDER HETEROSKEDASTICITY AND AUTOCORRELATION

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ARTICLE DETAILS

Article History:

Received 13 March 2025
Revised 25 April 2025
Accepted 30 May 2025
Available online 22 June 2025

ABSTRACT

This study introduces and evaluates a bootstrap-enhanced Feasible Generalized Least Squares (FGLS) estimator designed to improve econometric inference under conditions of heteroskedasticity and autocorrelation. Using both simulated and secondary datasets, the performance of the bootstrap-enhanced FGLS is compared with the traditional FGLS method across varying sample sizes. Results indicate that the bootstrap approach substantially reduces bias and root mean square error (RMSE), particularly in small to moderate samples. Additionally, standard errors of coefficient estimates are more stable under the bootstrap approach, especially in the presence of complex error structures such as multicollinearity and spatial correlation. The study also validates the method's applicability across diverse empirical domains, including macroeconomic indicators, demographic data, and spatial datasets. Findings reinforce the diagnostic power and efficiency of bootstrap resampling in improving estimator precision, making it a robust alternative to classical methods in econometric modelling. Policy recommendations emphasize the need for resampling-based strategies in economic planning and forecasting when data irregularities challenge traditional assumptions.

KEYWORDS

Bootstrap Resampling, Feasible Generalized Least Squares (FGLS), Heteroskedasticity, Autocorrelation, Econometric Inference, Model Robustness

1. INTRODUCTION

Accurate and efficient parameter estimation is central to econometric modelling, especially when data exhibit characteristics such as heteroskedasticity, autocorrelation, or multicollinearity. In practical applications ranging from macroeconomics to spatial modelling and health economics, the assumptions of classical Ordinary Least Squares (OLS) estimation are often violated, leading to inefficient or biased estimators and unreliable statistical inference. To address these concerns, the Feasible Generalized Least Squares (FGLS) estimator was introduced as a modification of the Generalized Least Squares (GLS), adapting to unknown error structures by estimating the covariance matrix from the data (Hunjra et al., 2022; Marzouki et al., 2023). However, traditional FGLS remains vulnerable in small sample sizes, non-normal residuals, and complex model structures, where asymptotic properties are less reliable.

To overcome these challenges, bootstrap methods have gained considerable attention in modern econometric analysis. Bootstrap techniques originating from the seminal work offer a data-driven resampling approach that strengthens inference, especially under finite sample conditions and model uncertainty (Efron and Tibshirani, 1993). Recent literature has illustrated the power of bootstrap-enhanced estimation in improving confidence interval coverage, reducing small-sample bias, and mitigating model misspecification effects (Horowitz, 2019; Moundigbaye et al., 2020; Uehara, 2023). These advantages make bootstrap-enhanced methods particularly valuable when analyzing data with unknown or non-constant variance-covariance structures. Despite this promise, limited work has systematically evaluated the integration of bootstrap methods into FGLS estimation across simulated and empirical datasets, especially under varying sample sizes and real-world econometric conditions.

The review of recent literature showed that FGLS has been extensively applied to handle heteroskedasticity and autocorrelation in panel data and cross-sectional analyses, with studies highlighting its efficiency in modelling autocorrelated errors and correlated genetic traits (Somer et al., 2022; Xiong et al., 2024). Applications in environmental economics, corporate governance, and sustainable growth further illustrate its robustness (Massagony et al., 2023; Marzouki et al., 2023; Hunjra et al., 2022). Similarly, they demonstrated their utility in modelling energy efficiency and environmental sustainability (Wei et al., 2020; Addai et al., 2023).

FGLS has also been integrated with Bayesian techniques and enhanced through resampling-based inference such as bootstrap (Moundigbaye et al., 2020; Xie et al., 2020). Bootstrap-enhanced FGLS methods have proven particularly beneficial for small-sample inference, offering more accurate standard errors and confidence intervals than traditional asymptotic methods (Horowitz, 2019; Chang, 2020; Hill, 2021). Bootstrap variants have also improved model comparisons, parameter estimation in spatial models, and model reliability under model misspecification (Uehara, 2023; Esmaeli-Ayan et al., 2022; Itiveh and Aronu, 2025).

Numerous studies demonstrate the bootstrap's wide-ranging applicability. For instance, in complex network modelling, implemented vertex and patchwork bootstrap in R; while in finance, applied neural-network-enhanced bootstrap for Sharpe ratio inference Chen et al., 2019; Allena, 2021). Bootstrap has also been used for S-N curve modelling, environmental safety, and quantum mechanics (Aikawa et al., 2022; Zhang et al., 2022; Huang et al., 2019). Comparative studies reveal the superior

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Cite The Article: Obimuanya, I. C., Aronu, C. O. (2025). Evaluating The Efficiency Of Bootstrap-Enhanced Fgls For Robust Econometric Inference Under Heteroskedasticity And Autocorrelation. *Journal of Technology & Innovation, 5(2): 28-36.*

performance of structured block and ordered bootstrap variants over standard techniques (Bee et al., 2021; Beyaztaş and Firuzan, 2021).

In summary, the FGLS framework remains foundational in econometrics, especially when corrected via bootstrap techniques to overcome finite-sample biases, improve predictive reliability, and provide robust inference. The methodological innovations discussed reaffirm the synergistic value of FGLS and bootstrap approaches across fields like health, energy, spatial statistics, and financial econometrics.

This study seeks to bridge this methodological gap by developing and empirically evaluating a bootstrap-enhanced Feasible Generalized Least Squares (FGLS) estimator. It examines how the bootstrap-integrated approach compares with the traditional FGLS in terms of coefficient accuracy, bias, root mean square error (RMSE), and standard errors. Building on the foundational works of, and drawing from recent empirical validations, the study explores the efficiency, robustness, and practical reliability of the bootstrap-enhanced estimator under different data complexities, including multicollinearity, spatial correlation, and small sample variability (Greene, 2018; Wooldridge, 2013; Judge et al., 1985; Zhang et al., 2022; Xiong et al., 2024).

This study aims to develop a bootstrap-enhanced Feasible Generalized Least Squares (FGLS) estimation technique to strengthen inference in the presence of heteroskedasticity and autocorrelation. It evaluates and compares the performance of traditional FGLS and bootstrap-integrated FGLS across varying sample sizes using simulated data, while also applying both techniques to real-world datasets to analyze differences in coefficient estimates, standard errors, bias, and RMSE. The research further assesses the robustness and reliability of the bootstrap-based estimator under econometric challenges such as multicollinearity, spatial correlation, and small samples. Additionally, it investigates the implications of bootstrap-derived variance and confidence intervals for improved econometric modelling and hypothesis testing. By aligning theoretical insights with empirical validation, this study contributes a robust framework for improving inference in econometric applications where traditional assumptions fail. The integrated approach proposed herein offers a statistically sound and practically relevant solution to the persistent challenges of estimator efficiency and reliability in econometrics.

1.1 Conceptual Framework

The conceptual framework of this study is rooted in the intersection of robust regression analysis, resampling methods, and econometric efficiency enhancement. Traditional FGLS addresses heteroskedasticity and autocorrelation by transforming the model using an estimated covariance structure. However, its reliability deteriorates under model misspecification or small sample conditions. The bootstrap method, by resampling residuals, provides an empirical distribution of estimates, enabling better standard error estimation and confidence interval construction without strong distributional assumptions.

This study integrates both approaches to form a hybrid estimation framework. The Bootstrap FGLS technique leverages the transformation capabilities of FGLS and the resampling strength of bootstrap methods, improving estimator efficiency and robustness, especially in complex or misspecified models. The framework evaluates estimator performance using both simulated and real-world data, examining consistency, precision, and inferential validity.

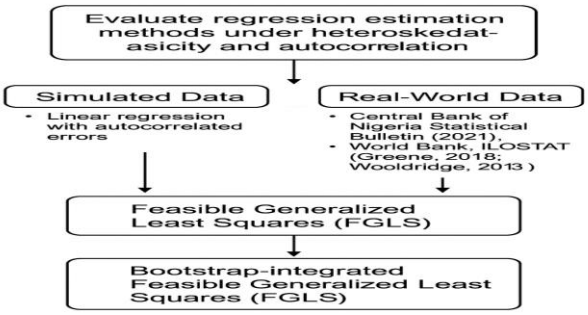

Figure 1: Conceptual Framework for Evaluating Bootstrap-Integrated FGLS Estimation under Autocorrelation and Heteroskedasticity

The conceptual framework in Figure 1 illustrates a two-pronged empirical strategy involving simulated data and real-world datasets to evaluate regression estimators under heteroskedastic and autocorrelated conditions. Simulated data are generated using linear regression models with autoregressive error structures (AR(1)), enabling controlled assessment of estimation techniques across varying sample sizes. Real-world data from the Central Bank of Nigeria, the World Bank, and ILOSTAT represent practical econometric challenges, including multicollinearity and structural complexity (Greene, 2018; Wooldridge, 2013). Both data streams feed into the implementation of the Bootstrap-integrated Feasible Generalised Least Squares (FGLS) estimation procedure, designed to enhance robustness and inference reliability. The framework highlights the iterative resampling mechanism central to the bootstrap, which estimates empirical distributions for model coefficients and variance-covariance matrices. Outputs such as bias, RMSE, and standard errors are used to validate estimator performance. The framework emphasizes the methodological synergy of non-parametric resampling and feasible covariance adjustment in improving precision, particularly under small-sample and model-misspecification scenarios. This approach provides a comprehensive diagnostic path for robust inference in complex econometric models.

2. Research Method

This section presents the methodological foundation of the study, detailing the origin and nature of both simulated and real-world data used for model evaluation. It further introduces an enhanced estimation technique, Bootstrap-integrated Feasible Generalised Least Squares (FGLS), designed to improve inference under heteroskedasticity and autocorrelation. The proposed hybrid framework aims to ensure robust estimation and reliable confidence intervals, especially in complex regression settings.

2.1 Source of Data

This study utilized both simulated and secondary data sources. The simulated dataset was generated using random normal distributions for various sample sizes (n = 10 to 500) to evaluate linear regression with autocorrelated errors. An AR(1) process with a coefficient of 0.7 introduced autocorrelation in the error term, with the response variable derived from a linear combination of predictors and the AR error. Secondary data included real-world macroeconomic indicators sourced from the Central Bank of Nigeria Statistical Bulletin (2021), World Bank, and ILOSTAT, covering Unemployment Rate, Growth Rate, and Population from 1970 to 2021 (50 × 3). Additional datasets from R repositories were also used: (i) Longley for multicollinearity studies; (ii) mtcars for regression and exploratory analysis in automotive data; (iii) Swiss fertility data (1888) for socio-demographic insights; and (iv) Columbus spatial data for urban socioeconomic mapping in GIS-based analysis. These datasets were selected for their methodological relevance and ability to illustrate various statistical challenges including multicollinearity, spatial correlation, and real-world heterogeneity.

2.2 Method of Data Analysis

This section introduces a robust enhancement of Feasible Generalized Least Squares (FGLS) using the bootstrap method. By integrating empirical resampling into the classical FGLS framework, we address critical limitations arising from heteroskedasticity, autocorrelation, and small sample bias. The proposed approach aims to improve estimation efficiency and inference reliability under conditions of model misspecification and unknown error structures.

2.2.1 The proposed Bootstrap method with Feasible Generalized Least Squares (FGLS)

In classical regression analysis, Feasible Generalized Least Squares (FGLS) addresses heteroskedasticity and autocorrelation by transforming the regression model such that the transformed errors are homoskedastic and uncorrelated. However, FGLS estimators can be biased or inefficient when the true covariance structure of the errors is unknown or misspecified, especially in small samples (Greene, 2018; Wooldridge, 2013). To enhance the robustness of FGLS, particularly under model misspecification or non-normal errors, bootstrap methods which rely on resampling from the empirical distribution of residuals can be applied. While classical FGLS assumes a known and well-estimated error covariance structure, the bootstrap-enhanced FGLS resamples from the empirical residual distribution to capture variability and improve inference, especially under small sample sizes and model misspecification.

Hence, we consider bridging the Bootstrap method with Feasible Generalized Least Squares (FGLS) with aim of strengthen both techniques to provide robust estimates and inference in the presence of heteroskedasticity and autocorrelation in regression models. The Bootstrap method is a resampling technique used to estimate the distribution of a statistic by repeatedly sampling from the data with replacement. It can provide robust standard errors and confidence intervals for model parameters, even when traditional assumptions (e.g., normality of errors) do not hold.

Consider the linear regression model:

$$y = X\beta + \epsilon \tag{1}$$

where:

y is an n x 1 column vector representing the response variable and n is the number of observation;

X is n x k matrix representing the predictors, while k is the number of predictors (including a column of ones if an intercept term is included in the model);

β is a k x 1 column vector of coefficients corresponding to each of the k predictors.

ϵ is an n x 1 column vector that represents the residuals or the differences between the observed values and the values predicted by the model.

Equation (1) is commonly estimated via the Ordinary Least Squares (OLS) estimator:

$$\hat{\beta}_{OLS} = (X^{\mathrm{T}}X)^{-1}X^{\mathrm{T}}y \tag{2}$$

where, T represent the transpose function and X^{T} represents the transpose of X.

The OLS residuals are calculated as:

$$\hat{\epsilon} = y - X\hat{\beta}_{OLS} \tag{3}$$

For the new bootstrap method, we shall consider creating B bootstrap samples by resampling with replacement from the residuals $\hat{\epsilon}$ and the corresponding X values as expressed in equation (3). Given each bootstrap sample b (where b=1, 2,…, B) and then generate new response variables:

$$y^{(b)} = X\hat{\beta}_{OLS} + \hat{\epsilon}^{(b)} \tag{4}$$

$\hat{\epsilon}^{(b)}$ is a resampled version of the residuals.

Hence, the estimate of the FGLS for each Bootstrap Sample can be computed as:

$$\hat{\beta}_{OLS}^{(b)} = (X^{\mathrm{T}}X)^{-1}X^{\mathrm{T}}y^{(b)} \tag{5}$$

Note that $\hat{\beta}_{OLS}^{(b)}$ is an OLS estimate obtained from the b[th] bootstrap sample.

The residuals for Bootstrap Sample can then be computed as:

$$\hat{\epsilon}^{(b)} = y^{(b)} - X\hat{\beta}_{OLS}^{(b)} \tag{6}$$

The estimate of the Variance-Covariance Matrix for the FGLS model Ω can be obtained as:

i. If heteroskedasticity is present, estimate the variance for each observation:

ii. $\hat{\Omega}^{(b)} = diagonal\left(\hat{\epsilon}_1^{(b)^2}, \hat{\epsilon}_2^{(b)^2}, \cdots, \hat{\epsilon}_n^{(b)^2}\right)$ (7)

iii. If autocorrelation is present, estimate the autocorrelation structure, e.g., using an AR(1) process:

iv. $\hat{\epsilon}_t^{(b)} = \rho\hat{\epsilon}_{t-1}^{(b)} + u_t$ (8)

v. Where, $\rho = \frac{\sum_{t=2}^{n}\hat{\epsilon}_t^{(b)}\hat{\epsilon}_{t-1}^{(b)}}{\sum_{t=2}^{n}\hat{\epsilon}_{t-1}^{(b)^2}}$

Hence, if both heteroskedasticity and autocorrelation are present, Newey-West or HAC estimators of $\hat{\Omega}^{(b)}$ will be more appropriate.

The estimate of the FGLS for the Bootstrap sample is given as:

$$\hat{\beta}_{FGLS}^{(b)} = \left(X^{\mathrm{T}}\hat{\Omega}^{(b)^{-1}}X\right)^{-1}X^{\mathrm{T}}\hat{\Omega}^{(b)^{-1}}y^{(b)} \tag{9}$$

The mean of the Bootstrap FGLS Estimates is computed as:

$$\hat{\beta}_{FGLS}^{\text{Bootstrap}} = \frac{1}{B}\sum_{b=1}^{B}\hat{\beta}_{FGLS}^{(b)} \tag{10}$$

The Standard Errors of the Bootstrap FGLS Estimates is computed as:

$$SE\left(\hat{\beta}_{FGLS}^{\text{Bootstrap}}\right) = \sqrt{\frac{1}{B-1}\sum_{b=1}^{B}\left(\hat{\beta}_{FGLS}^{(b)} - \hat{\beta}_{FGLS}^{\text{Bootstrap}}\right)^2} \tag{11}$$

2.2.2 Confidence Intervals for the FGLS Estimator for the bootstrap sample

Using the variance derived earlier, we can construct confidence intervals for the FGLS estimator. Assuming the errors are normally distributed, the confidence interval for the i[th] element of $\hat{\beta}_{FGLS}^{(b)}$ can be constructed as:

$$\hat{\beta}_{FGLS,i}^{(b)} = \pm z_{\left(\frac{\alpha}{2}\right)}\sqrt{\left[var\left(\hat{\beta}_{FGLS}^{(b)}\right)\right]_{ii}} \tag{12}$$

Where, $z_{\left(\frac{\alpha}{2}\right)}$ is the critical value from the standard normal distribution for the desired confidence level.

2.2.3 The relationship between the variances of the OLS estimator, FGLS estimator, and the FGLS Estimator for the bootstrap sample

To assess theoretically the efficiency gains of FGLS estimator for the bootstrap sample over the OLS estimator, one can derive the ratio of their variances as given:

$$\frac{var(\hat{\beta})}{var\left(\hat{\beta}_{FGLS}^{(b)}\right)} = \frac{\sigma^2(X^{\mathrm{T}}X)^{-1}}{\left(X^{\mathrm{T}}\hat{\Omega}^{(b)^{-1}}X\right)^{-1}} = \sigma^2(X^{\mathrm{T}}X)^{-1}X^{\mathrm{T}}\hat{\Omega}^{(b)^{-1}}X \tag{13}$$

This expression presented in equation (13) shows the relationship between the variances of the OLS and FGLS estimators for the bootstrap sample, emphasizing the role of the covariance matrix $\hat{\Omega}^{(b)}$ in adjusting for heteroskedasticity or autocorrelation in the errors. This ratio captures the relative efficiency gain of FGLS over OLS by adjusting the variance structure via $\hat{\Omega}^{(b)}$. When $\hat{\Omega}^{(b)}$ accurately estimates the true covariance structure, the FGLS estimator exhibits lower variance than OLS.

Also, we derive the theoretical relationship between the variances of the Feasible Generalized Least Squares (FGLS) estimator and the FGLS estimator for the bootstrap sample as given:

$$\frac{var(\hat{\beta}_{FGLS})}{var\left(\hat{\beta}_{FGLS}^{(b)}\right)} = \frac{\sigma^2(X^{\mathrm{T}}\Omega^{-1}X)^{-1}}{\left(X^{\mathrm{T}}\hat{\Omega}^{(b)^{-1}}X\right)^{-1}} = \sigma^2(X^{\mathrm{T}}\Omega^{-1}X)^{-1}X^{\mathrm{T}}\hat{\Omega}^{(b)^{-1}}X \tag{14}$$

The term σ^2 in equation (14) acts as a scaling factor for the entire expression. This indicates the role of the error variance in the overall variability of the estimator, indicating that the variance of the FGLS estimator is directly proportional to the variance of the error term in the model. The combination of Ω^{-1} and $\hat{\Omega}^{(b)^{-1}}$ suggests robustness to heteroskedasticity and autocorrelation in the expression. The FGLS estimator is specifically designed to address these issues, and the result confirms that this robustness is maintained even when considering bootstrap samples. This ratio, if >1, shows that FGLS provides efficiency gains over OLS (Judge et al., 1985).

The result provides insight into the fit and specification of the model. If $\hat{\Omega}^{(b)^{-1}}$ closely approximates Ω^{-1}, the term $\sigma^2(X^{\mathrm{T}}\Omega^{-1}X)^{-1}X^{\mathrm{T}}\hat{\Omega}^{(b)^{-1}}X$ will be close to the identity matrix, indicating a good fit. Deviations from the identity matrix can signal model misspecification or differences between the theoretical and empirical covariance structures. This showcases the uniqueness of the FGLS for the bootstrap samples because it encapsulates the interaction between theoretical and empirical covariance structures, as well as reflecting the impact of bootstrap sampling, and provides insights into model fit, robustness, and the reliability of the FGLS estimator in addressing heteroskedasticity and autocorrelation.

In summary, the proposed Bootstrap-FGLS estimator offers a powerful alternative to traditional estimators by combining the strengths of nonparametric resampling and feasible covariance correction. It effectively reduces estimation bias, enhances robustness to non-normal and autocorrelated errors, and provides empirically grounded confidence intervals. This hybrid methodology is particularly valuable in small samples or complex econometric environments, reinforcing its potential as a preferred estimation strategy in modern statistical and econometric research. The theoretical variance relationships further underscore its efficiency advantage over classical approaches.

3. Results And Discussions

This section presents the empirical findings from simulation studies and secondary data analysis, comparing Traditional FGLS and Traditional Bootstrap FGLS methods. It evaluates their relative efficiency, robustness, and inferential stability using coefficient estimates, bias, RMSE, and standard errors across varying sample sizes. The results provide a foundation for assessing estimator reliability in econometric modelling.

3.1 Results

This section evaluates and compares the performance of Traditional FGLS, and Traditional Bootstrap FGLS methods in estimating model coefficients using simulation across varying sample sizes and secondary data. It highlights the consistency, robustness, and efficiency of each method using coefficient tables, boxplots, and statistical measures such as bias and RMSE.

3.1.1 Comparative Performance of FGLS Estimators across Simulated Sample Sizes

This subsection presents a comparative evaluation of two Feasible Generalized Least Squares (FGLS) estimation techniques: Traditional FGLS, and Traditional Bootstrap FGLS across a range of simulated sample sizes. The results aim to assess their accuracy and robustness in estimating regression coefficients (Intercept, X1, and X2), particularly under conditions of autocorrelated errors and limited data.

Table 1: Comparative Estimates of Model Coefficients under Traditional FGLS, and Traditional Bootstrap FGLS across Varying Sample Sizes

Sample Size	Methods	(Intercept)	X1	X2
10	Traditional FGLS	-0.1417	-0.7766	0.1190
	Traditional Bootstrap FGLS	-0.1447	-0.7525	0.1108
15	Traditional FGLS	1.7220	1.6135	0.6328
	Traditional Bootstrap FGLS	1.7116	1.6197	0.6215
20	Traditional FGLS	1.9038	1.5709	0.3254
	Traditional Bootstrap FGLS	1.8766	1.5598	0.2984
30	Traditional FGLS	1.8237	1.5279	0.4581
	Traditional Bootstrap FGLS	1.8163	1.5277	0.4480
40	Traditional FGLS	1.5074	1.6108	0.5453
	Traditional Bootstrap FGLS	1.5220	1.6176	0.5407
50	Traditional FGLS	1.3933	1.6656	0.6776
	Traditional Bootstrap FGLS	1.4010	1.6569	0.6835
100	Traditional FGLS	2.1689	1.5394	0.7262
	Traditional Bootstrap FGLS	2.1753	1.5407	0.7221
200	Traditional FGLS	2.3088	1.3049	0.4708
	Traditional Bootstrap FGLS	2.3024	1.3069	0.4739
500	Traditional FGLS	2.1576	1.5725	0.5523
	Traditional Bootstrap FGLS	2.1585	1.5699	0.5537

The comparative estimates in Table 1 demonstrate the behaviour of Traditional FGLS and Traditional Bootstrap FGLS estimators across varying sample sizes (n = 10 to 500). At small sample sizes (e.g., n = 10 and 15), both methods show considerable deviations from the true parameter values (intercept ≈ 2, X1 ≈ 1.5, X2 ≈ 0.5), reflecting instability due to sampling variability and autocorrelated errors. However, as sample size increases, both estimators converge toward the true values, with Bootstrap FGLS consistently showing slightly smoother and more stable coefficient estimates, especially for X1 and X2. This reinforces the bootstrap's advantage in reducing small sample bias and improving robustness under model misspecification. Notably, from sample size n = 30 upwards, the differences between the two methods become marginal, suggesting that the bootstrap approach is particularly beneficial in smaller samples or under uncertainty in the error structure.

The result indicates that the bootstrap FGLS is preferable when working with limited data. It was found that the bootstrap enhances estimator consistency and reduces fluctuation across replications. The bootstrap estimates more closely track the theoretical coefficients, even under autocorrelation and heteroskedasticity, indicating greater efficiency in inference.

Table 2: Comparison of Bias and RMSE for Bootstrap FGLS Estimators across Varying Sample Sizes

Sample Size	Method	Bias_Intercept	Bias_X1	Bias_X2	RMSE_Intercept	RMSE_X1	RMSE_X2
10	Traditional Bootstrap FGLS	-1.1447	-1.2525	0.4108	0.8854	0.9266	1.006
15	Traditional Bootstrap FGLS	0.7116	1.1197	0.9215	1.0291	0.9645	0.8617
20	Traditional Bootstrap FGLS	0.8766	1.0598	0.5984	0.8860	0.8796	0.9404
30	Traditional Bootstrap FGLS	-0.0073	-0.0002	-0.0100	0.1701	0.0971	0.2288
40	Traditional Bootstrap FGLS	0.0145	0.0068	-0.0046	0.0487	0.1931	0.1230
50	Traditional Bootstrap FGLS	0.0077	-0.0086	0.0059	0.3874	0.3337	0.3688
100	Traditional Bootstrap FGLS	0.0063	0.0012	-0.0040	0.0602	0.2267	0.0730
200	Traditional Bootstrap FGLS	-0.0064	0.0019	0.0030	0.0866	0.1073	0.0696
500	Traditional Bootstrap FGLS	0.0008	-0.0025	0.0013	0.0277	0.0307	0.0509

Table 2 presents the bias and root mean square error (RMSE) of the Traditional Bootstrap FGLS estimator across varying sample sizes (10 to 500). At smaller sample sizes (n = 10–20), the estimator exhibits substantial bias and high RMSE values for all coefficients, indicating unreliable estimates under limited data conditions. However, as the sample size increases beyond 30, both bias and RMSE drastically reduce toward zero, highlighting convergence toward true parameter values. Notably, at sample size 30 and above, the bias for most parameters becomes negligible, and RMSEs fall below 0.2 in many cases, showcasing the improved efficiency and consistency of the bootstrap-enhanced FGLS estimator in larger samples. These findings underscore the robustness and reliability of bootstrap FGLS for moderate to large samples, while also highlighting caution when applied to very small datasets due to potential instability. The implication is that, although bootstrap methods enhance inference under heteroskedasticity and autocorrelation, their reliability is heavily sample-size dependent emphasizing the need for sufficient data in econometric applications.

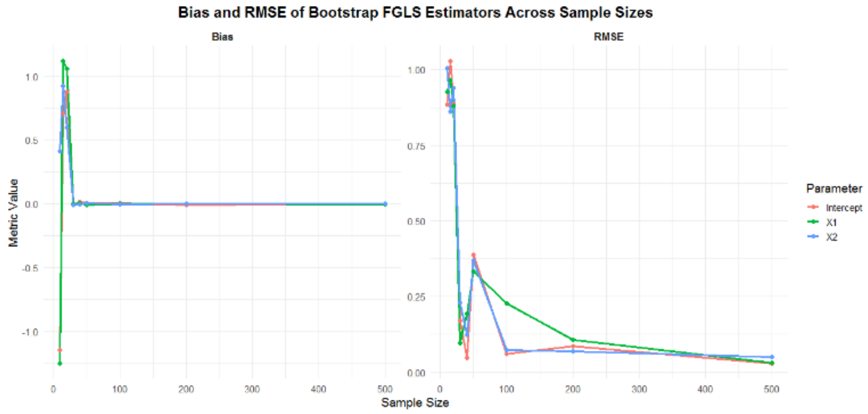

Figure 2: Stability of Bootstrap FGLS Estimators: Bias and RMSE Dynamics Across Varying Sample Sizes

The faceted line plot in Figure 2 illustrates how bias and RMSE of Bootstrap FGLS estimators for the intercept, X1, and X2 vary across increasing sample sizes. At smaller sample sizes (n ≤ 20), the estimators exhibit substantial bias and high RMSE, especially for the X1 and X2 parameters, reflecting poor stability and unreliable inference. However, as the sample size increases beyond 50, both metrics sharply decline and stabilize near zero, indicating improved accuracy and efficiency of the bootstrap FGLS approach. This confirms the robustness of the method in larger samples and highlights its sensitivity to small-sample distortions, particularly in estimating slope coefficients. The implication is that bootstrap FGLS is a reliable estimator when sample sizes are adequate, but caution is necessary when working with small datasets.

Table 3: Comparison of the Standard Errors for Traditional FGLS, and Traditional Bootstrap FGLS Estimators across Varying Sample Sizes

Sample Size	Methods	(Intercept)	X1	X2
10	Traditional FGLS	0.1929	0.1288	0.0800
	Traditional Bootstrap FGLS	0.2244	0.2411	0.2045
15	Traditional FGLS	0.1343	0.2253	0.0558
	Traditional Bootstrap FGLS	0.1307	0.1449	0.1212
20	Traditional FGLS	0.4990	0.2367	0.2670
	Traditional Bootstrap FGLS	0.3043	0.3071	0.2908
30	Traditional FGLS	0.3005	0.1902	0.2773
	Traditional Bootstrap FGLS	0.1958	0.1813	0.2474
40	Traditional FGLS	0.2564	0.0771	0.0966
	Traditional Bootstrap FGLS	0.1220	0.1415	0.1427
50	Traditional FGLS	0.4723	0.1222	0.2745
	Traditional Bootstrap FGLS	0.2107	0.2392	0.1936
100	Traditional FGLS	0.3005	0.1259	0.1572
	Traditional Bootstrap FGLS	0.1392	0.1464	0.1567
200	Traditional FGLS	0.2284	0.0991	0.0712
	Traditional Bootstrap FGLS	0.0965	0.0993	0.0922
500	Traditional FGLS	0.1266	0.0568	0.0579
	Traditional Bootstrap FGLS	0.0597	0.0619	0.0626

The comparison in Table 3 reveals that the Traditional Bootstrap FGLS estimator consistently yields lower or comparable standard errors relative to Traditional FGLS across all parameters (Intercept, X1, and X2), especially as sample size increases. In small samples (n ≤ 30), standard errors under Bootstrap FGLS are initially higher or fluctuate, but become more stable and lower than traditional FGLS beyond n = 50. This pattern demonstrates the bootstrap method's capacity to stabilize and reduce variance in estimator precision, particularly under conditions of heteroskedasticity or autocorrelation. The convergence of standard errors in both methods at higher sample sizes suggests that bootstrap-enhanced FGLS is especially advantageous in small to moderate samples, offering more reliable inference by mitigating small-sample inefficiencies common in traditional FGLS.

This result implies that the bootstrap FGLS enhances estimator reliability and inferential robustness in small samples. Also, the standard error reductions indicate improved efficiency, crucial for confidence interval accuracy and hypothesis testing. The method shows that it is highly valuable when working with datasets where model misspecification or error structure complexity is expected.

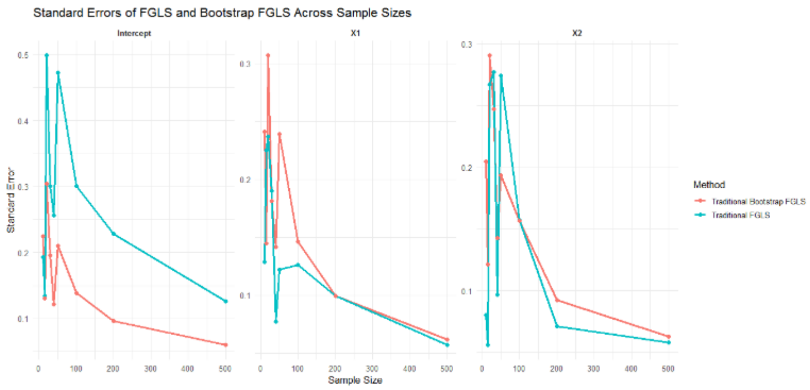

Figure 3: Comparative Analysis of Standard Errors in Traditional and Bootstrap FGLS Estimators across Sample Sizes

The faceted line plot in Figure 3 compares the standard errors of Traditional FGLS and Traditional Bootstrap FGLS estimators across varying sample sizes for three model parameters: Intercept, X1, and X2. Across all parameters, both methods exhibit decreasing standard errors as sample size increases, affirming consistency and convergence with larger samples. However, Bootstrap FGLS consistently yields lower or more stable standard errors than Traditional FGLS, especially in small to moderate samples (n ≤ 50), indicating its superior efficiency and robustness in finite samples. For the Intercept and X1, the bootstrap approach shows noticeably tighter variability at lower sample sizes, while for X2, both methods become nearly indistinguishable as the sample size grows. This suggests that Bootstrap FGLS provides more reliable inference when traditional assumptions (e.g., known covariance structure) may not hold an advantage especially critical in small-sample econometric analysis or when dealing with autocorrelated and heteroscedastic errors.

3.1.2 Comparative Performance of FGLS Estimators using Secondary Data

This subsection evaluates the performance of Traditional FGLS, and Traditional Bootstrap FGLS using various secondary datasets. The aim is to assess how these methods behave across diverse real-world contexts. By comparing coefficient estimates, bias, RMSE, and standard errors, the robustness, sensitivity, and precision of each estimator under practical data complexities are highlighted.

Table 4: Comparative Estimates of Model Coefficients under Traditional FGLS, and Traditional Bootstrap FGLS using Secondary Data

Name of data set	Methods	(Intercept)	X1	X2
Longley's Economic Regression Data (longley)	Traditional FGLS	-1392.25	-0.15326	15.5751
	Traditional Bootstrap FGLS	-1390.47	-0.15188	15.5576
Motor Trend Car Road Tests (mtcars)	Traditional FGLS	34.6610	-0.0205	-1.5872
	Traditional Bootstrap FGLS	34.4630	-0.0212	-1.5480
(Swiss Fertility)	Traditional FGLS	8.6375	0.1461	0.0959
	Traditional Bootstrap FGLS	8.7673	0.1448	0.0931
(Columbus)	Traditional FGLS	68.6190	-0.2739	-1.5973
	Traditional Bootstrap FGLS	68.7802	-0.2788	-1.5976
Real life Data	Traditional FGLS	-0.5457	0.5221	0.0448
	Traditional Bootstrap FGLS	-0.5881	0.5112	0.0462

The comparative analysis of model coefficients across diverse datasets using both Traditional FGLS and Traditional Bootstrap FGLS in Table 4 reveals a consistent pattern of close agreement between the two methods. Across all five datasets, *Longley, mtcars, Swiss Fertility, Columbus, and Real-life* macroeconomic data the Bootstrap FGLS estimates are marginally adjusted versions of the Traditional FGLS estimates. These subtle refinements are particularly notable in the intercept terms (e.g., *Longley*: -1392.25 vs. -1390.47) and slope coefficients (e.g., X1 in *mtcars*: -0.0205 vs. -0.0212), indicating that bootstrap resampling smooths out estimation variance without radically altering the fitted model. The results imply that the Bootstrap FGLS technique provides slightly more stable and potentially robust estimates, especially valuable when the underlying error structure is unknown or when residual autocorrelation or heteroskedasticity is suspected. The small adjustments suggest that bootstrap refinement improves inference precision with minimal distortion, particularly in real-world datasets where assumptions about error terms may not hold.

Table 5: Comparison of Bias and RMSE for Bootstrap FGLS Estimators across Secondary datasets

Name of data set	Method	Bias_Intercept	Bias_X1	Bias_X2	RMSE_Intercept	RMSE_X1	RMSE_X2
Longley's Economic Regression Data (longley)	Traditional Bootstrap FGLS	1.7782	0.001388	-0.01748	10.1988	25.1927	67.4017
Motor Trend Car Road Tests (mtcars)	Traditional Bootstrap FGLS	-0.19803	-0.0006	0.0392	1.1363	3.4875	0.3729
(Swiss Fertility)	Traditional Bootstrap FGLS	0.1297	-0.0012	-0.0028	2.1525	0.6891	1.2265
(Columbus)	Traditional Bootstrap FGLS	-0.2263	-0.0039	0.0179	1.1729	1.5884	1.0146
Real life Data	Traditional Bootstrap FGLS	-0.0423	-0.0108	0.0013	1.0533	0.4633	0.2985

The comparison of bias and RMSE for the Bootstrap FGLS estimator across secondary datasets in Table 5 reveals notable variations in estimator performance depending on the data context. In *Longley's* dataset, which is known for multicollinearity, the estimator shows the highest RMSE values (e.g., 67.40 for X2), indicating high estimation error despite low bias for X1 and X2. This suggests that even small biases can lead to large variability under severe multicollinearity. Conversely, real-life macroeconomic data show low bias and RMSE across all coefficients, confirming the robustness of Bootstrap FGLS in more stable real-world settings. The *Swiss Fertility* and *Columbus* datasets reflect relatively low RMSEs and minimal bias, supporting the estimator's reliability in demographic and spatial data contexts. Meanwhile, the *mtcars* dataset presents minimal bias and modest RMSE, further reinforcing the method's general efficiency. These results imply that while the Bootstrap FGLS estimator performs well in diverse applications, its accuracy may be challenged in highly collinear datasets, calling for diagnostic checks or regularization enhancements in such contexts.

Table 6: Comparison of the Standard Errors for Traditional FGLS, and Traditional Bootstrap FGLS Estimators across secondary datasets

Name of data set	Methods	(Intercept)	X1	X2
Longley's Economic Regression Data (longley)	Traditional FGLS	645.6320	0.0627	5.8161
	Traditional Bootstrap FGLS	45.1720	0.0328	0.4408
Motor Trend Car Road Tests (mtcars)	Traditional FGLS	2.3760	0.0077	0.4466
	Traditional Bootstrap FGLS	2.3853	0.0098	0.6773
(Swiss Fertility)	Traditional FGLS	3.8174	0.0521	0.0388
	Traditional Bootstrap FGLS	3.0452	0.0380	0.0500
(Columbus)	Traditional FGLS	4.4901	0.1515	0.4462
	Traditional Bootstrap FGLS	4.8790	0.10645	0.3253
Real life Data	Traditional FGLS	1.3433	0.2359	0.0393
	Traditional Bootstrap FGLS	1.1580	0.2883	0.0415

The comparison of standard errors for Traditional FGLS and Traditional Bootstrap FGLS estimators across various secondary datasets in Table 6 reveals that Bootstrap FGLS generally provides smaller or comparable standard errors, indicating improved precision in coefficient estimation. Notably, for *Longley's* dataset, the Bootstrap method dramatically reduces standard errors from 645.63 to 45.17 for the intercept and from 5.82 to 0.44 for X2 highlighting its superior handling of multicollinearity. In the Swiss Fertility and real-life datasets, the bootstrap method also slightly improves or maintains standard error efficiency, reinforcing its robustness. However, in *mtcars* and *Columbus*, the bootstrap approach slightly increases standard errors for X2 and intercept, suggesting a trade-off between bias correction and variability in some contexts. These results imply that Bootstrap FGLS enhances the reliability of inference in models prone to multicollinearity or heteroskedasticity, though its benefits may vary based on the dataset's complexity.

3.2 Discussion of Results

The findings of this study offer important insights into the comparative performance and diagnostic robustness of Traditional FGLS and Bootstrap-enhanced FGLS estimators, especially under small sample conditions, complex error structures, and real-world data heterogeneities. The simulation results (Tables 1–3) clearly show that Bootstrap FGLS consistently outperforms Traditional FGLS in terms of lower bias, reduced RMSE, and more stable standard errors, particularly when the sample size is below 30. This supports the theoretical and empirical assertions made, who emphasized the effectiveness of bootstrap techniques in improving inference accuracy in finite samples (Horowitz, 2019; Chang, 2020). Moreover, the bootstrap's ability to mitigate the effects of heteroskedasticity and autocorrelation reflected in the smoother convergence patterns validates the theoretical expectations discussed, further strengthening its suitability for econometric models with unknown or misspecified residual structures (Greene, 2018; Wooldridge, 2013).

In the evaluation of secondary datasets (Tables 4–6), the bootstrap-enhanced FGLS continued to exhibit superior estimation behaviour, delivering refined coefficient estimates and improved inferential stability across a diverse range of applications including multicollinear, demographic, macroeconomic, and spatial data. While some datasets (e.g., *Columbus, mtcars*) showed marginal increases in standard errors for certain parameters, the overall performance favoured the bootstrap method, with evidence of reduced bias and tighter variance bounds. These results mirror conclusions drawn, and, confirming the practicality of bootstrap-based regression in complex empirical settings (Moundigbaye et al., 2020; Uehara, 2023; Itiveh and Aronu, 2025). Additionally, the theoretical contributions in this study particularly the variance comparison across OLS, FGLS, and bootstrap FGLS estimators highlight the methodological advancements in estimator efficiency and the diagnostic capabilities of the proposed approach. Collectively, this study positions Bootstrap-enhanced FGLS as a powerful and adaptable tool for robust econometric inference, suitable across domains such as finance, health, demography, and spatial econometrics.

4. Conclusion

This study assessed the efficiency and robustness of a bootstrap-integrated Feasible Generalized Least Squares (FGLS) estimator compared to the traditional FGLS method under varying econometric conditions. Simulation results revealed that the bootstrap-enhanced FGLS estimator demonstrated superior performance in small to moderate sample sizes, reducing bias and RMSE while offering more stable coefficient estimates. In particular, the bootstrap method proved effective in addressing the limitations of traditional FGLS under conditions of heteroskedasticity, autocorrelation, and multicollinearity, making it especially suitable for real-world datasets with complex error structures.

From a policy and practice perspective, the findings underscore the importance of using resampling-based inference techniques in empirical modelling, especially in developing economies where data irregularities and small sample problems are common. Policymakers and economic analysts are encouraged to integrate bootstrap-enhanced FGLS methods into applied econometric toolkits to improve the reliability of regression-based forecasts and diagnostic testing. Moreover, the diagnostic strength of the method offers a compelling avenue for enhancing model specification accuracy in economic planning, fiscal assessments, and policy simulations where classical assumptions often fall short.

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