ZIBELINE INTERNATIONAL™ PUBLISHING

ISSN: 2773-6202 (Online)
CODEN: JTIOCC

Journal of Technology & Innovation (JTIN)

DOI: http://doi.org/10.26480/jtin.02.2024.75.78

REVIEW ARTICLE

UNDERSTANDING COBB-DOUGLAS PRODUCTION FUNCTION IN AGRICULTURAL ECONOMICS

Sristy Gautam*

Department of Agricultural Economics and Agribusiness Management, Faculty of Agriculture, Agriculture and Forestry University, Rampur Chitwan.
****Corresponding author Email: sristygtm@gmail.com***

ARTICLE DETAILS

Article History:

Received 23 June 2024
Revised 09 July 2024
Accepted 12 August 2024
Available online 14 August 2024

ABSTRACT

Cobb- Douglas production function shows the relationship between physical capital, labor inputs and the output produced. It is based on an empirical study of the American manufacturing industry conducted by Paul H. Douglas and C. W in 1928. The various modification and derivation of this production function is used in various analysis. It is a useful tool to agricultural economists. It can be used in estimation of production relationships, analyzing returns to scale, impact of technological change, policy analysis, sustainability, environmental impact and farm size efficiency analysis. It is popular because its form is relatively easy to use and is simple to parameterize through the use of regression analysis and correlation. It makes simple to determine the indicators of productivity and return on assets, the output elasticity for all parameters, and the marginal rates of substitution by the use of this function. Further improvements and several modifications to this model will probably increase its relevance and applicability in facing the future issues in agricultural economics as the farming practices and technologies continue to change.

KEYWORDS

Cobb-Douglas Production Function, Agricultural Economics, Production Relationships, Farm Efficiency Analysis

1. Introduction

The production function has been developed in order to understand and measure the relationship between the inputs used in the production process and the output generated (Gordon, 2011). To ensure efficient use of resources, farmers and firms need to have some understanding of the principles of production (Onalan and Basegmez, 2018). Cobb- Douglas production function shows the relationship between physical capital and labor inputs and the output produced (Besanko and Braeutigam, 2010). It is based on an empirical study of the American manufacturing industry conducted (Felipe, 2005). It was a significant milestone in macroeconomic modeling (Mishra, 2007; Felipe, 2005). The Cobb-Douglas production function is a very famous and accessible type of production function and is used widely in agriculture industry (Besanko and Braeutigam, 2010).

This review article's goal is to offer a thorough methodical analysis of the Cobb-Douglas production and provide insights on its uses, applications and properties. It will examine the relationship between agricultural inputs and output (Chambers, 1989; Mundlak, 2000; Heady and Dillon, 1961). The paper will also examine theoretical foundation and several aspects of this popular mathematical model with respect to agricultural economics (Vasyl'yeva, 2021). It will first describe the historical development of the Cobb-Douglas production function and examine its origins. It will also look at the presumptions that underlie this function. Case studies will be shown to highlight real-world applications. Implications for agricultural productivity analysis will also be covered. Possible drawbacks and criticisms of the function will also be analyzed.

2. Methodology

An extensive literature search and analysis are employed to conduct this article review. Initially, the articles containing the specified keywords were searched in the internet. Relevant scholarly databases and academic journals were systematically searched. Google scholar, Science direct, Scopus, etc. were used as search engines. Relevance of the topic and credibility of the authors and journals were the criteria for selecting articles for this collection. Review papers, Conference paper, reports and their abstracts were thoroughly examined, relevant books and books chapters were studied. The key concepts, methodologies, and empirical findings were extracted after identifying potential articles. Subsequently, articles that did not meet the inclusion criteria were excluded, and the complete texts of all qualifying articles were thoroughly studied.

3. Review Findings

3.1 Concept

Production function is the technical relationship between input and output in production process (Shephard, 2015). It is a functional form of the production function, which represents the relationship between physical capital and labor inputs and the output produced (Ioan and Ioan, 2015). Cobb Douglas production function have been used by agricultural economists for the production process which involves conversion of inputs into output (Debertin, 2012).

3.2 Historical Development

In 1928, Cobb and Douglas used time series data and conducted their first investigation of the relationship between capital, labor and manufacturing output in the United States (Humphrey, 1997). The authors found that the observed results and their theoretical production function matched quite closely. Over the next few decades, a number of studies were carried out that verified the initial findings of Cobb and Douglas and provided empirical support for their model (Miller, 2008). Initially, the function proposed in the 1928 article was $y = Ax_1^{\alpha}x_2^{\alpha-1}$ where, x1 = labor and x2 = capital (Cobb and Douglas, 1928). It was developed collaboratively by Charles W. Cobb (mathematician) and Paul Douglas (economist) in 1928, building on earlier ideas from other economists (Woodworth, 1977). Paul

Quick Response Code	Access this article online	
	Website: www.jtin.com.my	**DOI:** 10.26480/jtin.02.2024.75.78

Douglas was looking for a functional form to relate his labor and capital estimates.

Douglas then consulted mathematician Charles Cobb, who suggested using the function $Y = AL^{\beta}K^{(1-\beta)}$, where Y is output, A represents technology, L is labor, K is capital, and β is the output elasticity of labor, which was previously used by economists such as Knut Wicksell, Philip Wicksteed, and Léon Walras (Sandelin, 1976). The breakthrough for the Cobb-Douglas function occurred when Douglas used US census data, which provided a large number of cross-sectional observations, to present the results during his 1947 speech as president of the American Economic Association and it was popularized in the US by Paul H. Douglas in an article in 1928 (Debertin, 1986; Just and Pope, 1979).

3.3 Mathematical Formulation

The Cobb-Douglas production function is given as:

$$Q = AL^{\alpha}K^{\beta}$$

where, L = real value of labor input,

K = real value of capital input,

Q = real value added in output,

α = output elasticity with respect to labor, and

β = output elasticity with respect to capital (Bhanumurthy, 2002).

The equation indicates that output depends directly on L and K and that part of output that cannot be explained by L and is explained by A, which is often called technical change. The coefficient of laborer in the C-D function measures the percentage increase in Q that would result from a one-percent increase in L, while keeping K constant. Similarly, β represents the percentage increase in Q that would arise from a one percent increase in K, while maintaining L as constant (Smirnov and Wang, 2021).

3.4 Properties

Some properties of a Cobb-Douglas production function are as follows:

3.4.1 Functional Form

Cobb-Douglas production function takes the form: $Q = A * L^{\alpha} * K^{\beta}$, where, L = real value of labor input, K = real value of capital input, Q= real value added in output, α =output elasticity w.r.t. to labor, and β = output elasticity w.r.t. to capital (Bhanumurthy, 2002).

3.4.2 Output Elasticities

The output elasticities are shown by the power in the Cobb-Douglas production function. Here, α represents output elasticity with respect to labor and β represents output elasticity with respect to capital. It can be explained by using the formula to calculate the elasticity.

Output elasticity w.r.t labor (E_L)= $\frac{\partial Q}{\partial L} \times \frac{L}{Q}$

$$\text{Here, } \frac{\partial Q}{\partial L} = MP_L = A\alpha L^{\alpha-1}K^{\beta}$$

$$= \frac{\alpha A L^{\alpha}K^{\beta}}{L}$$

$$= \frac{\alpha Q}{L} \quad [Q = AL^{\alpha}K^{\beta}]$$

$$\text{Now, } \frac{\partial Q}{\partial L} \times \frac{L}{Q} = \frac{\alpha Q}{L} \times \frac{L}{Q}$$

$$= \alpha$$

Hence, Output elasticity w.r.t labor (E_L)= α

Similarly, Output elasticity w.r.t capital (E_K) = β (Brown, 2018)

3.4.3 Returns to Scale

The production function solved by Cobb-Douglas had a 1/4 contribution of capital to the increase in manufacturing industry and 3/4 of labor, so that the C-D production function is now solved: $Q = AL^{3/4}C^{1/4}$ (Mishra, 2007). This shows constant returns to scale because the total of the values of L and K is equal to one: (3/4 + 1/4), i.e., (α + β = 1). Here, α + β = returns to scale, and MC/MR= α + β. So, if returns to scale is constant, MC = MR (i.e. profit maximizes) (Smirnov, 2019). And, when α + β = 1 there are constants returns to scale (CRS) which shows that raising production levels won't boost output; the only way to boost economic efficiency is by raising technical standards. When α + β > 1 there are increasing returns to scale (IRS). It shows that using the current technology to expand production scale can lead to higher output. When α + β < 1 there are decreasing returns to scale (DRS) which shows that using the current technology to expand production scale in order to increase output is not worth the gain. This feature makes it possible to understand how variations in input levels impact output in terms of scale efficiency (Shephard, 2015).

3.4.4 Homogeneous Function

C-D production function is a homogeneous function, the degree of homogeneity of the function being α + β, which is equals to one. Hence, it is homogeneous production function of degree one. It implies that if the input is increased by certain factor, then output is also increased by the same factor (Debertin, 2012).

3.4.5 Diminishing Marginal Returns

The Cobb-Douglas function exhibits diminishing marginal returns to each input. It implies that as an additional input is added while maintaining the other inputs constant, the marginal product of that input decreases. It is the fundamental feature of the Cobb-Douglas model (Orolando, 2023). It can be proved as:

We know, $MP_L = \frac{\alpha Q}{L}$

Taking 2nd order derivative w.r.t labor

$$MP_L = \frac{\partial Q^2}{\partial L} = \frac{\partial}{\partial L} A\alpha L^{\alpha-1}K^{\beta}$$

$$= A\alpha(\alpha - 1)L^{\alpha-2}K^{\beta}$$

$$= \frac{\alpha(\alpha - 1)AL^{\alpha}K^{\beta}}{L^2}$$

We know that α+β=1, so α is less than 1. In the above equation, (α-1) will be negative which will make the whole term negative. And when the 2nd order derivate is negative, it will exhibit diminishing returns with respect to labor. We can derive similar equation for capital and it will also exhibit the same pattern of diminishing marginal returns (Brown, 2018).

3.4.6 Logarithmic transformation

The Cobb Douglas production function can be converted to log form and the parameters can be estimated using linear regression techniques.

$$Q = AL^{\alpha}K^{\beta}$$

Taking Log on both sides,

$$\log Q = \log A + \alpha \log L + \beta \log K + \mu$$

Where, μ is the error term (Praveen et al., 2019).

3.5 Assumptions

- All markets must have perfect competition, meaning that prices serve as parameters (Labini, 1995).
- The returns have to be constant in order for Euler's theorem to be applied and for the sum of the two exponents, α and β to be equal one (Labini, 1995).
- In order to treat the total capital as though it were a single good, it is assumed that the various capital goods are flexible and adaptable at will (Labini, 1995).
- The substitution of elasticity is taken as one i.e. constant (Labini, 1995).
- Total capital's worth can be calculated without taking return into account (Labini, 1995).

3.6 Applications in Agricultural Economics

3.6.1 Estimation of Production Relationships

The Cobb-Douglas production function can be used to estimate the elasticities of output with respect to inputs that helps in understanding relative contribution to agricultural production by each input (Sau, 1971). Marginal productivity of each input can also be estimated which is crucial for determining optimum resource use and for improvement of production efficiency (Chambers, 1989).

3.6.2 Analyzing Returns to Scale

The function can be used to estimate whether agricultural production exhibits increasing, decreasing or constant returns to scale. This feature makes it possible to understand how variations in input levels impact output in terms of scale efficiency and helps in the understanding of different farming practices and operational scale (Mundlak, 2000).

3.6.3 Impact of Technological Change

Cobb-Douglas function can also determine the impact of technological advancements on productivity by incorporating technological variables. This might be important in assessing the innovation (such as improved farming techniques, mechanized farming and genetically modified crops) effect on output (Heady and Dillon, 1961).

3.6.4 Policy Analysis

The function helps in evaluating the effectiveness of policies aimed at improving resource allocation such as subsidies, training programs and infrastructure development (Mundlak, 2000). It also evaluates the impact of agricultural policies on income distribution among different groups, particularly small-scale farmers and large-scale farmers (Heady and Dillon, 1961).

3.6.5 Sustainability and Environmental Impact

Input-output relationships and their environmental impact can be analyzed using this production function, which evaluates the sustainability of agricultural practices, including effects on biodiversity, water resources and soil health (Chambers, 1989).

3.6.6 Farm Size Efficiency

The function can be used to understand the dynamics and compare small and larger operations/farms in developing countries. It can also be used to determine the impact on the productivity and sustainability by farm size, labor availability, and capital (Mundlak, 2000).

3.7 Case Examples

Some researchers explored application of Cobb-Douglas production function to analyze agricultural economies in Poland (Parlińska and Dareev, 2011). The analysis revealed key insights into the contributions of capital and labor to agricultural productivity. In Poland, the function $X=1\cdot10-13K^{3.493}L^{0.0458}$ indicated high dependency on capital. 1% increase in fixed assets lead to 3.5% rise in gross output and labor's impact was found minimal**.** Similarly, a group researcher made a study in of coefficients of input factors in agricultural production in Sichuan Province China and estimated output elasticity to measure contribution rate of agricultural technological progress to growth of gross agricultural production (Wang, et al., 2021). He analyzed the impact of different input factors and the result showed that labor and material expense had a positive and significant effect while land had a negative and insignificant effect on agricultural output.

Another study was done on the application of the Cobb-Douglas production function in analyzing the effectiveness of productive resources in agricultural enterprises of primary production. The study focused on agricultural businesses in Nitra region of Slovakia during the transformative period from 1998 to 2001. The analysis aimed to evaluate efficiency of resource allocation (Kotulič and Pavelková, 2014). Similarly, research conducted by Tirfi and Oyekale in Ethiopia on showcasing the climate change adaptation to enhance maize yield used a modified Cobb Douglas production function (Tirfi and Oyekale, 2023). Climatic variables impact was analyzed on maize yield and it found that increase in seasonal rainfall and mean maximum temperature negatively affected yield while the minimum temperature showed a positive impact.

3.8 Advantage

The following are some of the benefits of the Cobb-Douglas production function:

- The Cobb-Douglas production function can describe the state of returns to scale regardless of whether they increase, are stable, or decrease (Smirnov and Wang, 2021).
- Its form is relatively easy to use, making it simple to determine the indicators of productivity and return on assets, the output elasticity for all parameters, and the marginal rates of substitution (Orolando, 2023).
- It is practical and universally applicable and adequate as the Cobb-Douglas production function coefficients directly describe the elasticity of each input factor used (Brown, 2018).
- It is based on actual economic indicators from official statistical reporting and is simple to parameterize through the use of regression analysis and correlation (Praveen et al., 2019).
- The result's functional dependence on costs is nonlinear and lacks the drawbacks of linear production functions that characterize (Vasyl'yeva, 2021).

3.9 Limitations and Criticisms

This is based on the assumption that production resources are fully interchangeable (Bhanumurthy, 2002).

The marginal prices of factors equal to average prices and calculated on the basis of market prices. This is possible in conditions of perfect competition and market, not in the real economy (Vasyl'yeva, 2021).

It is not applicable for a large number of inputs (Labini, 1995).

It is based on assumptions of perfect competition in the factor and product markets which is restrictive (Labini, 1995).

It assumes constant returns to scale (CRS) (Mundlak, 2000).

Serial correlation and heteroscedasticity are major problems (Bhanumurthy, 2002).

As the Labor and capital are correlated and the estimates are liable to be biased (Bhanumurthy, 2002).

It assumes unitary elasticity of substitution which is unrealistic (Debertin, 1986).

It is inflexible in form (Bhanumurthy, 2002).

Estimates derived from a single equation will inevitably be inconsistent (Smirnov, 2019).

4. Conclusion

The Cobb-Douglas production function offers a framework for understanding the relationship between input and output in agricultural production and is a vital tool in the field of agricultural economics. Mathematical representation of labor and capital contribution to the production facilitates the analysis of resource allocation efficiency, technological impacts and also the effectiveness of agricultural policies. The various modification and derivation of this production function is used in various analysis. Its relative ease of use makes it a powerful tool for agricultural economists. This also helps with the application of policies to achieve efficient and sustainable agricultural production and decision-making regarding the distribution of available resources. Though the Cobb-Douglas production function has ability to provide valuable insights into the production process, it has various constraints. Further improvements and several modifications to this model will probably increase its relevance and applicability in facing the future issues in agricultural economics as farming practices and technologies continue to change.

Conflicts of Interest

I declare that there are no conflicts of interest associated with this manuscript.

References

Besanko, D.A., and Braeutigam, R.R., 2010. Microeconomics (4th ed.). John Wiley and Sons.

Bhanumurthy, K.V., 2002. Arguing a case for the Cobb-Douglas production function. Review of Commerce Studies, 20-21 (1).

Brown, M., 2018. Cobb–Douglas functions. In The New Palgrave Dictionary of Economics (pp. 1738-1741). Palgrave Macmillan. https://doi.org/10.1057/978-1-349-95189-5_480

Chambers, R.G., 1989. Applied production analysis: A dual approach. Cambridge University Press.

Cobb, C. W., & Douglas, P. H. (1928). A Theory of Production. *The American Economic Review*, *18*(1), 139–165. http://www.jstor.org/stable/1811556

Dana, H., and Jaromír, H., 2007. Cobb-Douglas production function: The case of a converging economy. Czech Journal of Economics and Finance, 57 (9-10), Pp. 465-476.

Debertin, D.L., 1986. The Cobb Douglas production function. In Agricultural Production Economics (pp. 171-186). University of Kentucky.

Debertin, D.L., 2012. The Cobb-Douglas production function. In Agricultural Production Economics (pp. 171-186). Macmillan Publishing Company.

Diwan, R.K., 1968. On the Cobb Douglas production function. Southern Economic Journal, 34 (3), Pp. 410-414. https://doi.org/10.2307/1055504

Felipe, J., and Adams, F.G., 2005. A theory of production, the estimation of the Cobb-Douglas function: A retrospective view. Eastern Economic Journal, 31 (3), Pp. 427-445. https://www.jstor.org/stable/40326423

Gordon, R.V.D., 2011. The historical role of the production function in economics and business. American Journal of Business Education, 4 (4).

Heady, E.O., and Dillon, J.L., 1961. Agricultural production functions. Iowa State University Press.

Humphrey, T.M., 1997. Algebraic production functions and their uses before Cobb-Douglas. Economic Review, 83 (1), Pp. 51-83.

Ioan, A.C., and Ioan, G., 2015. The complete theory of Cobb-Douglas production function. Acta Universitatis Danubius, 11 (1), Pp. 74-114.

Just, R.E., and Pope, R.D., 1979. Production function estimation and related risk considerations. American Journal of Agricultural Economics, 61 (2), Pp. 276-284. https://doi.org/10.2307/1239732

Kotulič, R., and Pavelková, J., 2014. The application of the Cobb-Douglas production function in analyzing the effectiveness of productive resources in agricultural enterprises of primary production. Journal of Central European Agriculture, 15 (3), Pp. 284-301. https://doi.org/10.5513/jcea01/15.3.1489

Labini, P.S., 1995. Why the interpretation of the Cobb-Douglas production function must be radically changed. Structural Change and Economic Dynamics, 6 (4), Pp. 485-504. https://doi.org/10.1016/0954-349x(95)00025-i

Miller, E., 2008. An assessment of CES and Cobb-Douglas production functions. Congressional Budget Office.

Mishra, S.K., 2007. A brief history of production functions. North-Eastern Hill University Shillong (India), Department of Economics. https://doi.org/10.2139/ssrn.1020577

Mundlak, Y., 2000. Agriculture and economic growth: Theory and measurement. Harvard University Press.

Onalan, O., and Basegmez, H., 2018. Estimation of economic growth using grey Cobb-Douglas production function: An application for US economy. Journal of Business, Economics and Finance, 7 (2), Pp. 178-290. https://doi.org/10.17261/Pressacademia.2018.840

Orolando, G., 2023. On the assumptions of the Cobb-Douglas production function and their assessment in contemporary economic theory. Social Science Research Network.

Parlińska, M., and Dareev, G., 2011. Applications of production function in agriculture. Quantitative Methods in Economics, 12 (1), Pp. 119-123.

Phraser, I. (2002, March). The Cobb-Douglas production function: An Antipodean defence? Economic Issues, 7, 39-58.

Praveen, P.J., Donthi, R., Prasad, S.B., Mahaboob, B., and Venkateswarlu, B., 2019. A glance on the estimation of Cobb-Douglas production functional model. AIP Conference Proceedings, Pp. 2177. AIP Publishing. https://doi.org/10.1063/1.5135242

Sandelin, B., 1976. On the origin of the Cobb-Douglas production function. Economy and History, 19 (2), Pp. 117-123. https://doi.org/10.1080/00708852.1976.10418933

Sau, R.K., 1971. Agricultural revolution by production function. Economic and Political Weekly, 6 (26), Pp. A50-A58. https://www.jstor.org/stable/4382196

Shephard, R.W., 2015. Theory of cost and production functions. Princeton University Press.

Smirnov, R.G., 2019. The Cobb-Douglas production function revisited. In Recent Developments in Mathematical, Statistical and Computational Sciences: The V AMMCS International Conference (pp. 725-734). Springer, Cham. https://doi.org/10.1007/978-3-030-63591-6_66

Smirnov, R.G., and Wang, K., 2021. The Cobb-Douglas production function revisited. Springer Proceedings in Mathematics and Statistics. https://doi.org/10.1007/978-3-030-63591-6_66

Tirfi, A.G., and Oyekale, A.S., 2023. An Augmented Cobb-Douglas Production Function Modeling of the impact of climate change on maize yields in Ethiopia. Agro Ekonomi/Agro Ekonomi, 34 (1), Pp. 12. https://doi.org/10.22146/ae.76238

Vasyl'yeva, O., 2021. Assessment of factors of sustainable development of the agricultural sector using the Cobb-Douglas production function. Baltic Journal of Economic Studies, 7 (2), Pp. 37-49. https://doi.org/10.30525/2256-0742/2021-7-2-37-49

Wang, J., Song, H., Tian, Z., Bei, J., Zhang, H., Ye, B., and Ni, J., 2021. A method for estimating output elasticity of input factors in Cobb-Douglas production function and measuring agricultural technological progress. IEEE Access, 9, Pp. 26234-26250. https://doi.org/10.1109/access.2021.3056719

Woodworth, R.C., 1977. Agricultural production studies. University of Minnesota Press.